(11) Plane Geometry

1. In the figure (not drawn to scale), \( AE \) is the diameter of the circle, \( F \) is the mid-point of \( CE \) and \( ED \) is tangent to the circle at \( E \). \( CG, BF \) and \( AE \) are parallel lines.
   a) Prove that \( \triangle ACE \) is congruent to \( \triangle EDC \)
   b) Prove that \( \triangle CBF \) is similar to \( \triangle DEC \)
   c) Prove that \( CD \times CE = 4CF \times BF \)
   d) Prove that \( AE^2 = CA^2 + CD^2 + DE^2 \)

   ![Diagram for Plane Geometry 1](image)

2. In the figure, \( BD \) and \( CD \) are tangents to the circle at \( B \) and \( C \) respectively. \( AB \) is parallel to \( CD \) and \( BE \) is perpendicular to \( CD \).
   a) Prove that \( \triangle ABC \) is an isosceles triangle
   b) Prove that \( \triangle DCB \) is similar to \( \triangle CAB \)
   c) Prove that \( CE^2 + EB^2 = AB \times DB \)

   ![Diagram for Plane Geometry 2](image)
3. In the diagram, $CD$ is a tangent to the circle $ABC$ at $C$. $BC$ is a common chord between both circles and $F$ is the intersection between lines $AE$ and $CD$.
   a) Prove that $AC$ is parallel to $DE$.
   b) Prove that $CF^2 = AF \times FB$

4. In the diagram, $BD$ and $DF$ are tangents to the circle and $4ABD = 24DBC$. Prove that $AD = DF$.

5. $BCF$ is an equilateral triangle inscribed in a circle. $ABC$ is a tangent to the circle at point $B$. Given that $EF = HL$ and $AEGL$ is a straight line, prove that
6. In the diagram, the 2 circles intersect at points $D$ and $F$. $ABC$ is a tangent to the bigger circle at $B$ and $HFB$ is a tangent to the smaller circle at $F$. $ADF, EDB$ and $EFG$ are straight lines.

a) Prove that $BF = BG$.

b) Show that $ABC$ is parallel to $EFG$

c) Show that $\triangle BDA$ is similar to $\triangle EGB$

![Diagram for 6]

7. In the diagram, $B$ is the mid-point of $AC$ and $G$ is the mid-point of $AD$. $CDE$ and $BGF$ are straight lines. Given that $BH = 2HE$, show that,

a) $\triangle DHE$ is similar to $\triangle GHB$

b) $AH = 5DH$

![Diagram for 7]
8. The diagram shows triangle \( ABC \) where \( AD = DF \), \( AB = 2BE \) and \( BC = 3CD \). Find the value of \( \frac{\text{Area of } \triangle CDE}{\text{Area of } \triangle AEF} \).

\[ \frac{\text{Area of } \triangle CDE}{\text{Area of } \triangle AEF} \]

9. In the figure, \( ABCD \) is a square with sides 8 cm and \( AEFG \) is a rectangle. Given that \( AG = 6 \text{ cm} \), find the length of \( GF \).

10. In the diagram, \( AFE \) is a triangle inscribed in a circle. \( ABCD \) is a parallelogram and \( AB \) is a tangent to the circle at point \( A \). Prove that