Kinematics

1 The velocity, \( v \) m/s, of a particle moving in a straight line, \( t \) seconds after leaving a fixed point \( O \) is given by \( v = t^2 + kt + 12 \), where \( k \) is a constant. At \( t = 3 \) s the particle rests momentarily at point \( M \).

a) Find the other value of \( t \) where the particle is momentarily at rest.
b) Calculate the average speed of the particle for the first 6 seconds.
c) Calculate the time at which the particle passes point \( M \) again.

a) When \( t = 3, v = 0 \)
\[
0 = 3^2 + k(3) + 12
\]
\[
k = -7
\]
\[
\therefore v = t^2 - 7t + 12
\]
\[
0 = t^2 - 7t + 12
\]
\[
(t - 3)(t - 4) = 0
\]
The particle is momentarily at rest when \( t = 3 \) and when \( t = 4 \)

b) \( s = \int t^2 - 7t + 12 \, dt \)
\[
s = \frac{t^3}{3} - \frac{7t^2}{2} + 12t + c
\]
When \( t = 0, s = 0, \therefore c = 0 \)
\[
\therefore s = \frac{t^3}{3} - \frac{7t^2}{2} + 12t
\]
When \( t = 3, \)
\[
s = \frac{3^3}{3} - \frac{7(3)^2}{2} + 12(3)
\]
\[
s = 13.5m
\]
When \( t = 4, \)
\[
s = \frac{4^3}{3} - \frac{7(4)^2}{2} + 12(4)
\]
\[
s = 13.33 \, m
\]
When \( t = 6, \)
\[
s = \frac{6^3}{3} - \frac{7(6)^2}{2} + 12(6)
\]
\[
s = 18 \, m
\]
Total distance travelled = \( 13.5 + (13.5 - 13.33) + (18 - 13.33) \)
\[
= 18.34m
\]
Average speed = \( \frac{\text{total distance}}{\text{total time}} \)
\[
= \frac{18.34}{6}
\]
\[
= 3.06 \, m/s
\]

c) When \( s = 13.5, \)
\[
13.5 = \frac{t^3}{3} - \frac{7t^2}{2} + 12t
\]
\[
2t^3 - 21t^2 + 72t - 81 = 0
\]
By Observation, \( (t - 3) \) is a repeated root. Apply long division to divide by \( (t - 3)^2 \): \( 2t^3 - 21t^2 + 72t - 81 = (t - 3)^2(2t - 9) = 0 \)
\[
t = 3 \quad \text{or} \quad t = \frac{9}{2} = 4.5
\]
The particle passes \( M \) again at 4.5s
A particle moves in a straight line. After time $t$ seconds, the velocity of the particle (in m/s) is $v = 16 + 4t - kt^2$, where $k$ is a constant.

a) If the maximum velocity is 20 m/s, find the value of $k$.
b) Find the time when the particle is moving at its initial velocity again.

a) $v = 16 + 4t - kt^2$
   \[ \frac{dv}{dt} = 4 - 2kt \]
   \[ 0 = 4 - 2kt \]
   \[ t = \frac{4}{2k} = \frac{2}{k} \]

\[ 20 = 16 + 4 \left( \frac{2}{k} \right) - k \left( \frac{2}{k} \right)^2 \]
\[ 4 = \frac{8}{k} - \frac{4}{k} \]
\[ k = \frac{4}{4} = 1 \]

b) When $t = 0$, $v = 16$ m/s
   \[ 16 = 16 + 4t - t^2 \]
   \[ t(4 - t) = 0 \]
   \[ t = 0 \text{ (Rej)} \quad \text{or} \quad t = 4 \]
Two cyclists, Alvin and Bryan, are moving in the same direction on the same straight track. At a certain point $O$, Alvin is travelling at a speed of $20 \text{ m/s}$ and decelerate uniformly at $4 \text{ m/s}^2$, overtakes Bryan who is travelling at $4 \text{ m/s}$ and accelerating uniformly at $2 \text{ m/s}^2$.

a) Find the distance between Alvin and Bryan three seconds after passing $O$.

b) Calculate the velocity of Bryan when he overtakes Alvin.

a) Let $a_A, v_A, s_A$ be Alvin’s acceleration, velocity and displacement from $O$ respectively

Let $a_B, v_B, s_B$ be Bryan’s acceleration, velocity and displacement from $O$ respectively

$a_A = -4$

$v_A = \int -4 \, dt = -4t + c$

When $t = 0$, $v_A = 20$,

$v_A = -4t + c$

$20 = -4(0) + c$

$c = 20$

∴ $v_A = -4t + 20$

$s_A = \int -4t + 20 \, dt$

$s_A = -2t^2 + 20t + c$

When $t = 0$, $s_A = 0$, ∴ $c = 0$

∴ $s_A = -2t^2 + 20t$

When $t = 3$

$s_A = -2(3)^2 + 20(3) = 42 \text{ m}$

$a_B = 2$

$v_B = \int 2 \, dt = 2t + c$

When $t = 0$, $v_B = 4$,

$v_B = 2t + c$

$4 = 2(0) + c$

$c = 4$

∴ $v_B = 2t + 4$

$s_B = \int 2t + 4 \, dt$

$s_B = t^2 + 4t + c$

When $t = 0$, $s_B = 0$, ∴ $c = 0$

∴ $s_B = t^2 + 4t$

When $t = 3$

$s_B = (3)^2 + 4(3) = 21 \text{ m}$

Distance between Alvin and Bryan at three seconds $= 42 - 21 = 21\text{ m}$

b) When $s_A = s_B$

$-2t^2 + 20t = t^2 + 4t$

$3t^2 - 16t = 0$

$t = 0 \text{ or } t = \frac{16}{3} = 5 \frac{1}{3}$

When $t = 5 \frac{1}{3}$,

$v_B = 2 \left(5 \frac{1}{3}\right) + 4 = 14.7 \text{ m/s}$