(3) Polynomials

1. Without using long division, find the remainder when \(2x^6 + x^4 - 15x^2 - 14\) is divided by \(x^2 + 2\).

2. A cubic polynomial, \(f(x)\), leaves a remainder of 12 when divided by \(x\) and \(f(x + 1) - f(x - 1) \equiv 12x^2 - 12x - 42\). By substituting suitable values of \(x\),
   a) Find the remainder when \(f(x)\) is divided by \((x - 2)\)
   b) Show that \(f(-2) = 30\)
   c) Show that \((x - 4)\) is a factor of \(f(x)\).

3. Given that \((x - 1)(x - 2)(Ax + B) + C(x - 2) + D = 3x^3 - 7x^2 + 3x + 2\) for all values of \(x\), find \(A, B, C\) and \(D\).

4. \((x - 2)\) is a factor of \(g(x) + 5\), where \(g(x)\) is a polynomial. Find the remainder when \(f(x) = (2x^3 + 3x^2 - 4)g(x)\) is divided by \((x - 2)\).

5. The term containing the highest power of \(x\) in the polynomial \(f(x)\) is \(x^4\) and the roots of \(f(x) = 0\) are -6 and 3. \(f(x)\) has a remainder of -84 when divided by \((x - 1)\) and a remainder of -96 when divided by \((x - 2)\). Find the expression for \(f(x)\).

6. Express \(\frac{3x^2 + 5}{x^3 - 1}\) in partial fractions

7. Given that \(f(x) = 4x^3 - 2x^2 + 5x - 1\), find
   i) the remainder when \(f(x)\) is divided by \((x - 1)\)
   ii) the remainder when \(f(x - 8)\) is divided by \((x - 9)\).
   iii) deduce the remainder when \(f(x^2 - 6)\) is divided by \((x^2 - 8)\).

8. When the function \(f(x)\) is divided by \((x + 1)\), the remainder is -5. When \(f(x)\) is divided by \((x - 1)\), the remainder is -1. When \(f(x)\) is divided by \((x^2 - 1)\), the remainder is \((Ax + B)\).
   Find \(A\) and \(B\).

9. \(f(x)\) is a function where \(f(x) = ax^3 + bx^2 + 2x - 5\). \(2f(x) - 6\) is divisible by \((x - 1)\) and when \(f(x) + 4\) is divided by \((x + 2)\), it leaves a remainder of -5. Find \(A\) and \(B\).

10. Given that \((x^2 - 3)\) is a factor of \(f(x) = x^3 + ax^2 + bx - 3\)
    i) Find the value of \(a\) and \(b\).
    ii) Hence, factorize \(f(x)\) completely.
    iii) Hence, solve the equation \(1 + ay + by^2 - 3y^3 = 0\).