(1) Simultaneous, Quadrilaterals and Inequalities

1. Find the value(s) of \( k \) for the following simultaneous equations, given that the equations have no solution.
   \[
   (k + 1)y = (2k - 1)x + 5 \quad \text{---(1)}
   \]
   \[
   4y = (k + 2)x + 10 \quad \text{---(2)}
   \]

2. The equation \( 2x^2 + 8x = 1 \) has roots \( \alpha \) and \( \beta \).
   a) State the value of \( \alpha + \beta \) and \( \alpha\beta \)
   b) Find the value of \( \alpha^2 - \beta^2 \), leaving your answer in surd form.
   c) Find the quadratic equation whose roots are \( \alpha^4 - \beta^4 \)

3. It is given that \( \alpha \) and \( \beta \) are the roots of the equation \( y = x^2 - x - 1 \), where \( \beta > \alpha \) and that \( \alpha + \frac{1}{\alpha} \) and \( \beta + \frac{1}{\beta} \) are the roots of another quadratic equation with integer coefficients. Without solving the values of \( \alpha \) and \( \beta \), find the exact value of \( \alpha + \frac{1}{\alpha} \)

4. a) If one root of the equation \( 4x^2 - 22x + k = 0 \) is ten times the other, find the value of \( k \).
    b) Show that \( 2 - x^2 + 3x \) can never be greater than 5.

5. Show that the roots of the equation \( x^2 + (2 - k)x = \frac{3}{2}k \) are real for all real values of \( k \).

6. The roots of the equation \( x^2 - 4x + k \) differs by \( 2k \). Show that \( s^2 = 4 - k \). Given also that the roots are positive integers and that \( k \) is a positive integer, find the possible values of \( s \).

7. Given that \( \alpha \) and \( \beta \) are the roots of the equation \( x^2 = x - 5 \), prove that
   a) \( \frac{1 - \alpha}{5} = \frac{1}{\alpha} \)
   b) \( \alpha^3 + 4\alpha + 5 = 0 \)

8. a) Find the range of values of \( x \) for which \( 2x^2 + x - 6 \) lies between \(-3\) and 4.
    b) Show that if the roots of the equation \( 2x^2 + 3x - 2 + m(x - 1)^2 = 0 \) are real, then \( m \) cannot be greater than \( \frac{25}{12} \).

9. Find the range of values of \( k \) for which the graph of \( y = kx^2 - 3x + kx \) lies entirely above the line \( y = 4 \).

10. i) Show that the expression \( x^2 - x + \frac{7}{2} \) is always positive for all real values of \( x \).
     ii) Hence, find the values of \( k \) which satisfy the inequality \( \frac{-x^2 + kx + 2}{-(x^2 - x + 3.5)} < 2 \) for all real values of \( x \).

11. The roots of the equation \( 2x^2 - 8x + 50 = 0 \) are \( \alpha^2 \) and \( \beta^2 \). Find
     i) the value of \( \alpha^2 + \beta^2 \) and \( \alpha^2\beta^2 \).
     ii) two different quadratic equations whose roots are \( \alpha \) and \( \beta \)