Linear Law

1. The diagram shows part of a straight line obtained when plotting values of \( \ln(y + 2) \) against \( \ln(x + 1) \). Express \( y \) in terms of \( x \).

Gradient = \( \frac{5-3}{6-2} = \frac{1}{2} \)
Equation of line : \( Y = \frac{1}{2} X + c \)
Sub \((2,3)\) into equation:
\[ 3 = \frac{1}{2}(2) + c \]
\[ c = 2 \]
Equation of line : \( Y = \frac{1}{2} X + 2 \)
\( \ln(y + 2) = \frac{1}{2} \ln(x + 1) + 2 \)
\( \ln(y + 2) = \ln(x + 1)^{\frac{1}{2}} + \ln e^2 \)
\( \ln(y + 2) - \ln(x + 1)^{\frac{1}{2}} = \ln e^2 \)
\( \frac{y + 2}{\sqrt{x + 1}} = e^2 \)
\( y = e^2 \sqrt{x + 1} - 2 \)

2. In each of the following, \( a \) and \( b \) are unknown constants. Express each of them into the form \( Y = mX + c \), where \( X \) and \( Y \) are functions of \( x \) and/or \( y \), and \( m \) and \( c \) are constants.
\( y^b = 10^{x+a} \)

Ans: a) \( \lg y = \frac{1}{b} x + \frac{a}{b} \)
b) \( \lg y = (\lg a)x + \lg(b + 2) \)
c) \( \frac{1}{y^2} = \frac{1}{a^2} x - \frac{b}{a^2} \)
3. Alvin and Gina both used linear law to express the same equation into forms suitable for drawing straight line graphs. As they expressed the equation differently, 2 different graphs were obtained (as shown below). Determine the original equation relating $x$ and $y$.

Solution:

$y^2 = m(xy) + c$  

Substitute Coordinates into Equation (1):

$(6) = m(1) + c$  

Divide equation (1) by $y^2$:

$1 = m \left( \frac{x}{y} \right) + c \left( \frac{1}{y^2} \right)$  

Substitute Coordinates into Equation (3):

$1 = m(2) + c(-3.5)$  

Simultaneously solve (2) and (4):

$m = 4$
$c = 2$

$\therefore$ Equation is $y^2 = 4(xy) + 2$